Distributed MCMC Inference in Dirichlet Process Mixture Models Using Julia

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*Both these authors contributed equally.

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Outline

1. Motivation for Bayesian Nonparametric Mixture Models
2. Dirichlet Process Mixture Models (DPMMs)
3. Parallel MCMC Sampler for DPMMs [Chang & Fisher, NIPS '13]
4. Distributed & Parallel MCMC Sampler for DPMM [present work]
5. Results

Bayesian Nonparametric Mixture Models

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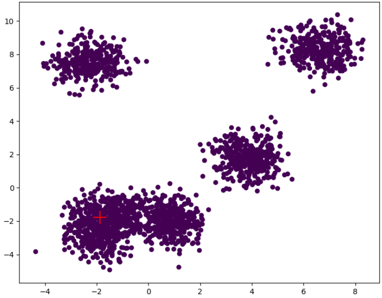

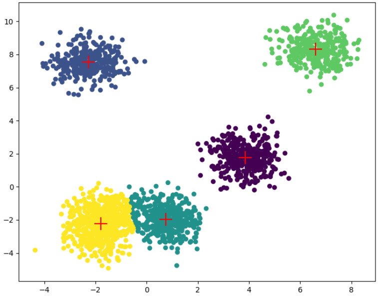

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- Problems:
  - Requires performing clustering many times (one for each value of K).
  - For each of value of K: the fitting often gets stuck in a poor local maximum.

Bayesian Nonparametric Mixture Models

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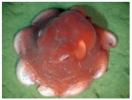
The approach:
Bayesian nonparametric mixture models

Applications

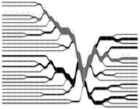
[wikipedia.org]



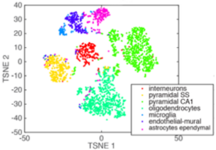
[Ed Bowlby, NOAA]

[Fox et al 2014]

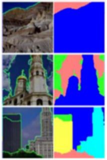
[Lloyd et al 2012; Miller et al 2010]



[Ewens 1972; Hartl, Clark 2003]

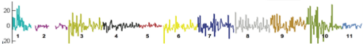


[Prabhakaran, Azizi, Carr, Pe'er 2016]

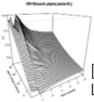


[Sudderth, Jordan 2009]

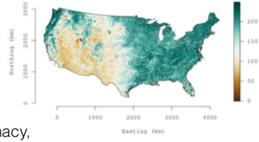
[Saria et al 2010]



[US CDC PHIL; Futoma, Hariharan, Heller 2017]

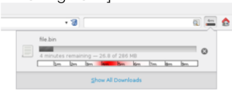


[Gramacy, Lee 2009]



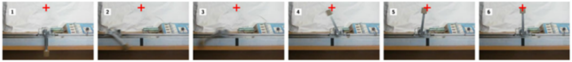
[Datta, Banerjee, Finley, Gelfand 2016]

[Kiefel, Schuler, Hennig 2014]



[Chati, Balakrishnan 2017]

[Deisenroth, Fox, Rasmussen 2015]



Slide: from Tamara Broderick's Tutorial on Bayesian Nonparametrics.

First, We Need Some Basic Concepts

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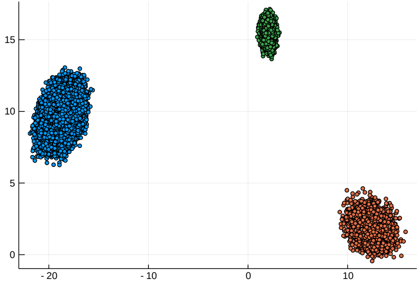
- The Chinese Restaurant Process (one construction of DP)
- Dirichlet Process Mixture Model (DPMM, [Escobar and West, 1995] [2])

Prior on components

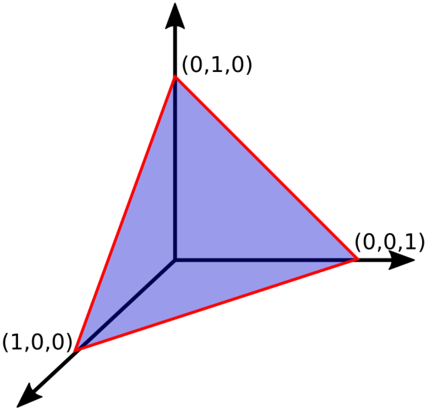
Every componenet has a weight. The weights can be:

- Known.
- Unknown and determinstic.
- Unkown and random.



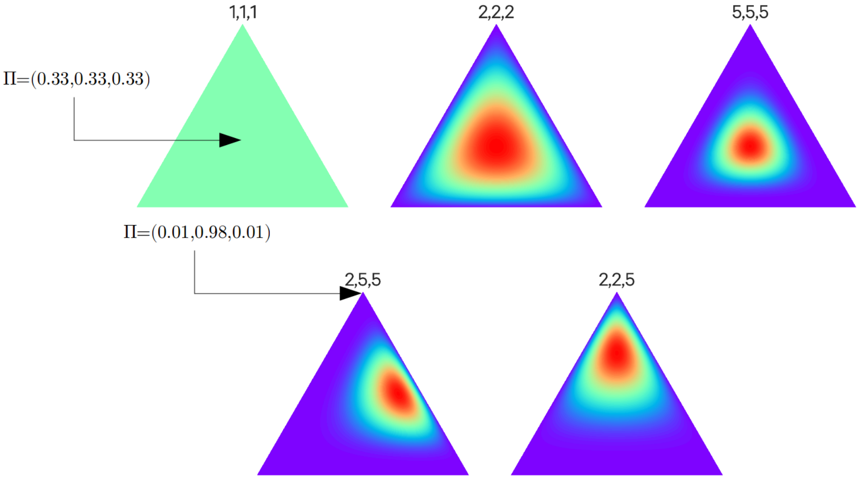
Dirichlet Distribution

$\mathrm{Dir}(\cdot)$ is a distribution over distributions.



Dirichlet Distribution

Examples for $\mathrm{Dir}(\alpha_1, \alpha_2, \alpha_3)$, $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$ is a point on the simplex.



Dirichlet Distribution

- $\boldsymbol{\pi} = \mathrm{Cat}(\pi_1, \pi_2, .., \pi_K)$ is a Categorical distribution.

$$\pi_j \in (0, 1), \quad \sum_{j=1}^{K} \pi_j = 1 \tag{1}$$

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- $\boldsymbol{\pi} \sim \mathrm{Dir}(\alpha_1, \alpha_2, ..., \alpha_K)$ is the probability to draw the distribution $\boldsymbol{\pi}$.

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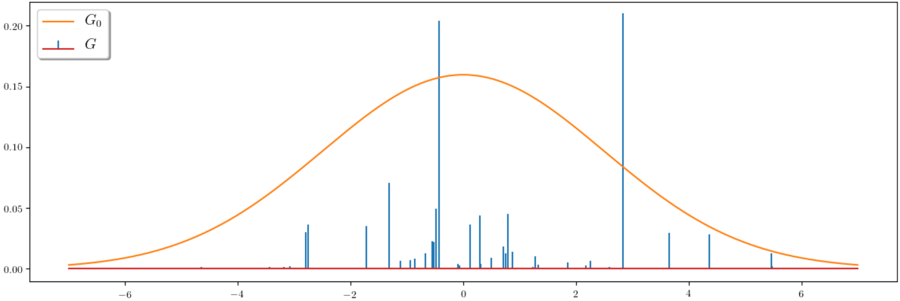
- $G \sim \mathrm{DP}(\alpha, G_0)$:
  - G_0 - Base probability measure, either continuous or discrete.
  - α - Concentration parameter.
  - G - Random probability measure, discrete.

Dirichlet Process - Example

$$G_0 = \mathcal{N}(0, 2.5) \quad G \sim \mathrm{DP}(\alpha = 10, G_0) \tag{2}$$



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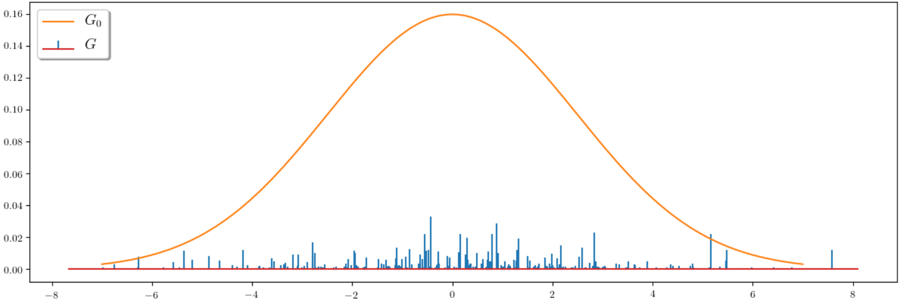

The Chinese Restaurant Process

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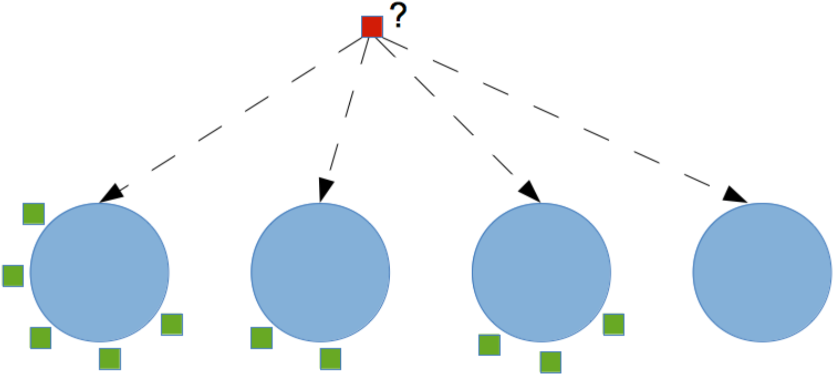
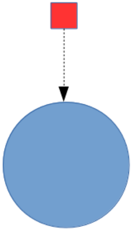
- At a restaurant with an infinite amount of tables, what is the chance for a new customer to sit at an existing table, or to open a new table?



The Chinese Restaurant Process

The first customer sits at the first table with probability 1.



The Chinese Restaurant Process

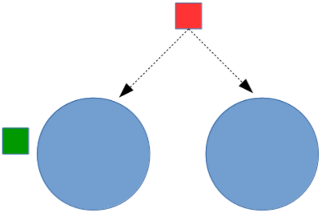
The second customer can either join an existing table with probability

$$p = \frac{|X_1|}{n - 1 + \alpha},$$

or open a new table with probability

$$p = \frac{\alpha}{n - 1 + \alpha}.$$

$|X_1|$ - Customers count at table 1.
α - Concentration parameter.
n - Customers count at the rest.



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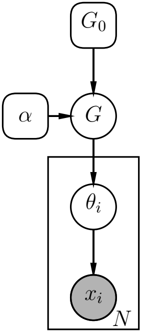

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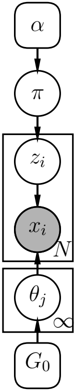

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- Remark: there also exist other efficient inference methods.

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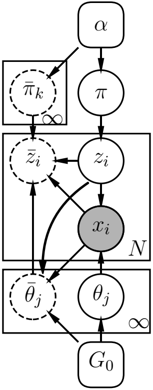

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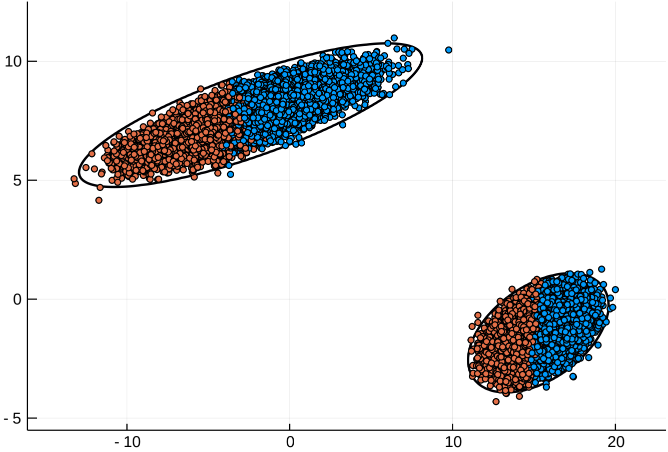

Visualization of the augmented space, 2 clusters, each has its points associated with either 'left' or 'right' sub-cluster.

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$$H_{\mathrm{split}} = \frac{\alpha \Gamma(N_{jl}) f_x(x_{\mathcal{I}_{jl}}; \lambda) \cdot \Gamma(N_{jr}) f_x(x_{\mathcal{I}_{jr}}; \lambda)}{\Gamma(N_j) f_x(x_{\mathcal{I}_j}; \lambda)} \tag{9}$$

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Large Moves

Merges/Splits allows us to do large moves, changing many labels at a time, and often allowing us to escape a local maximum.

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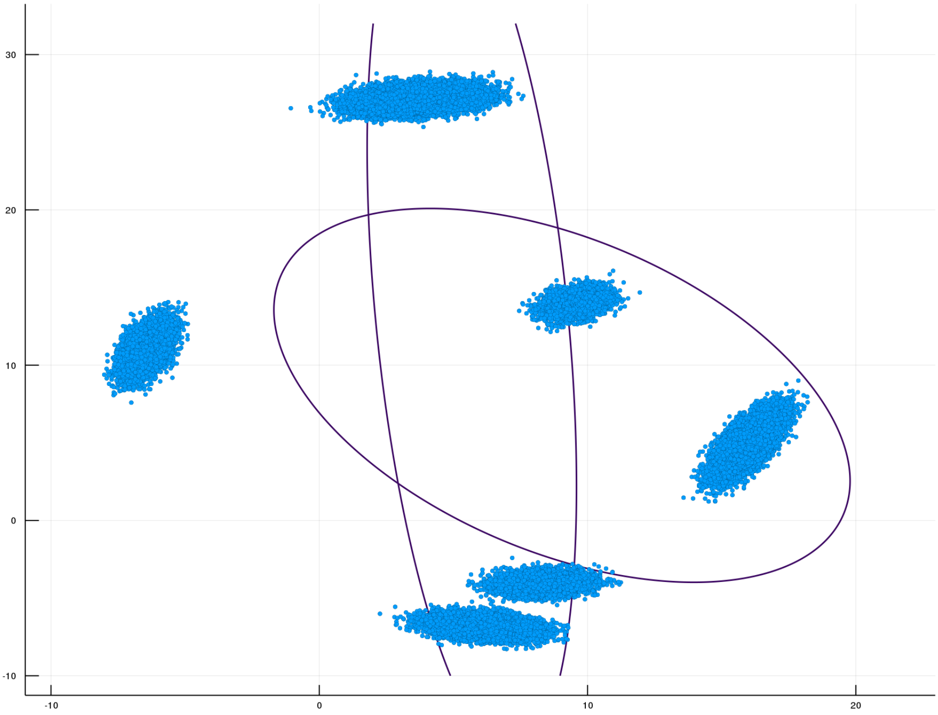

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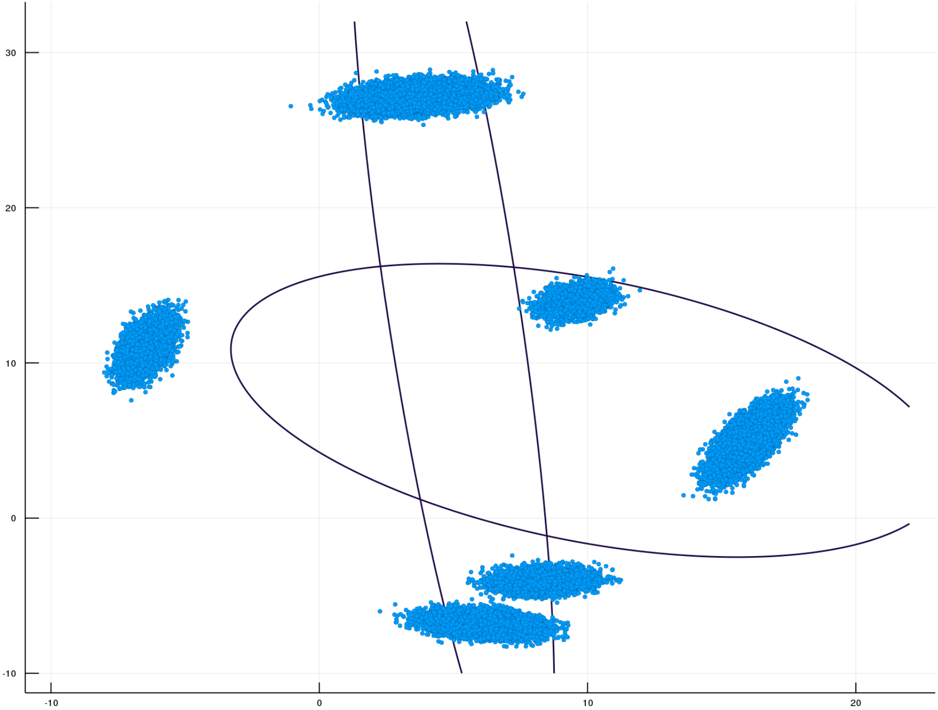

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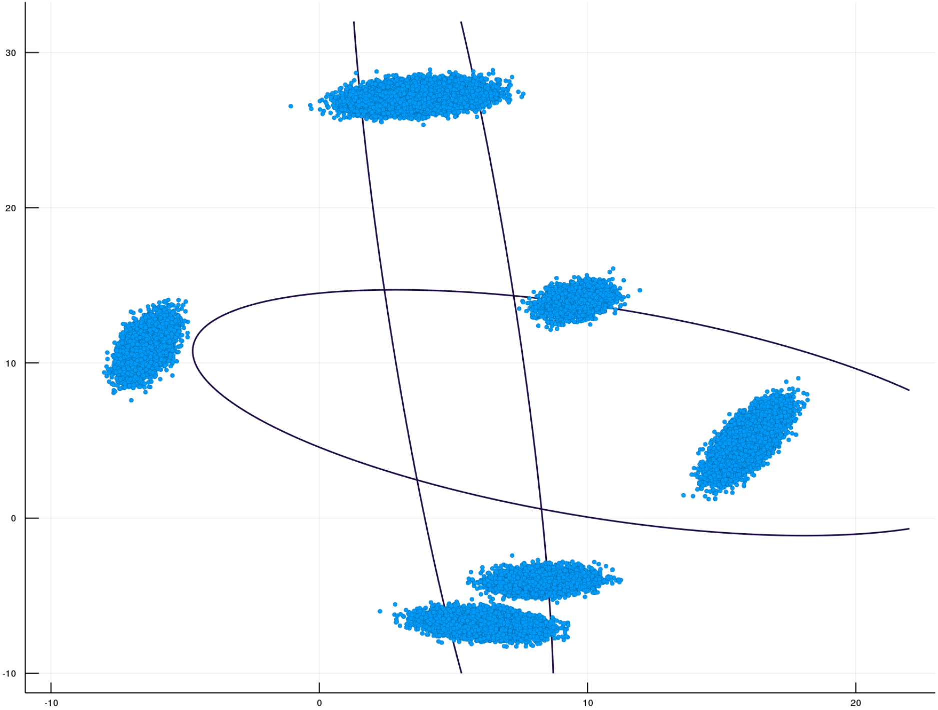

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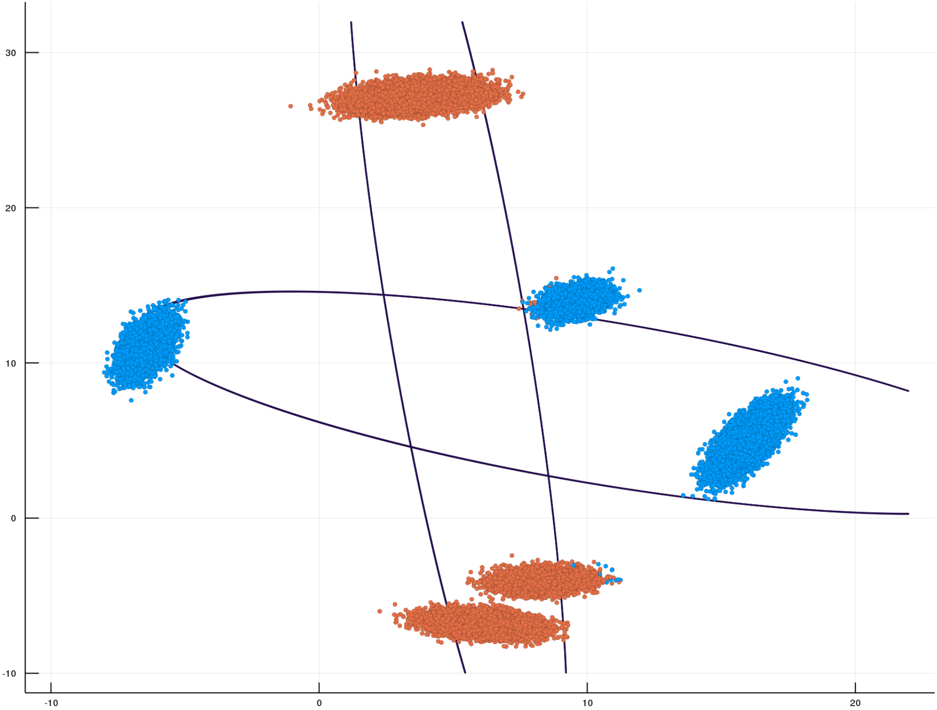

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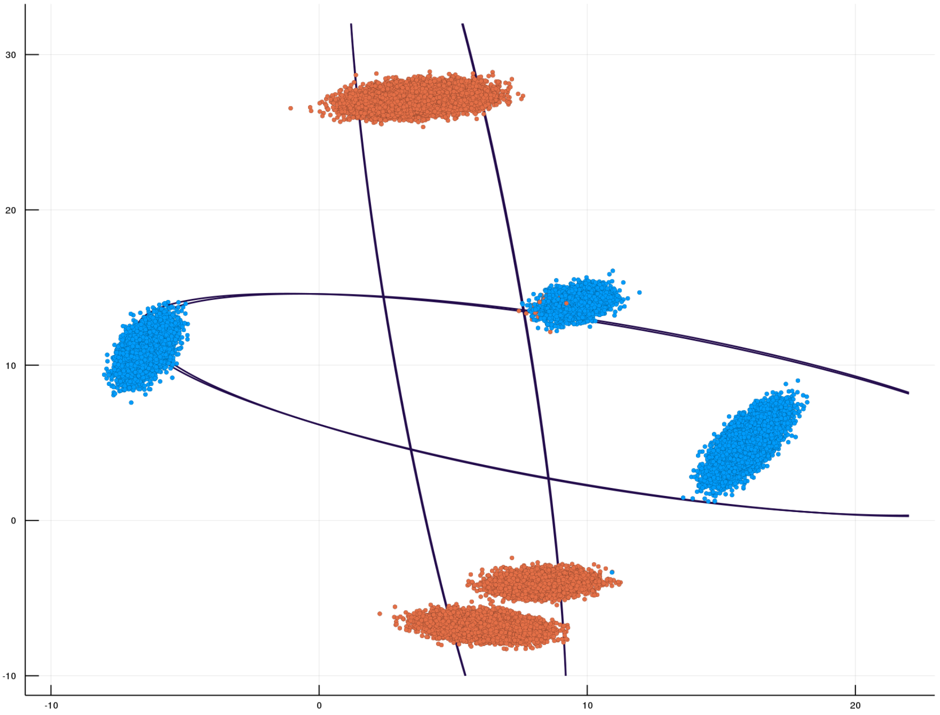

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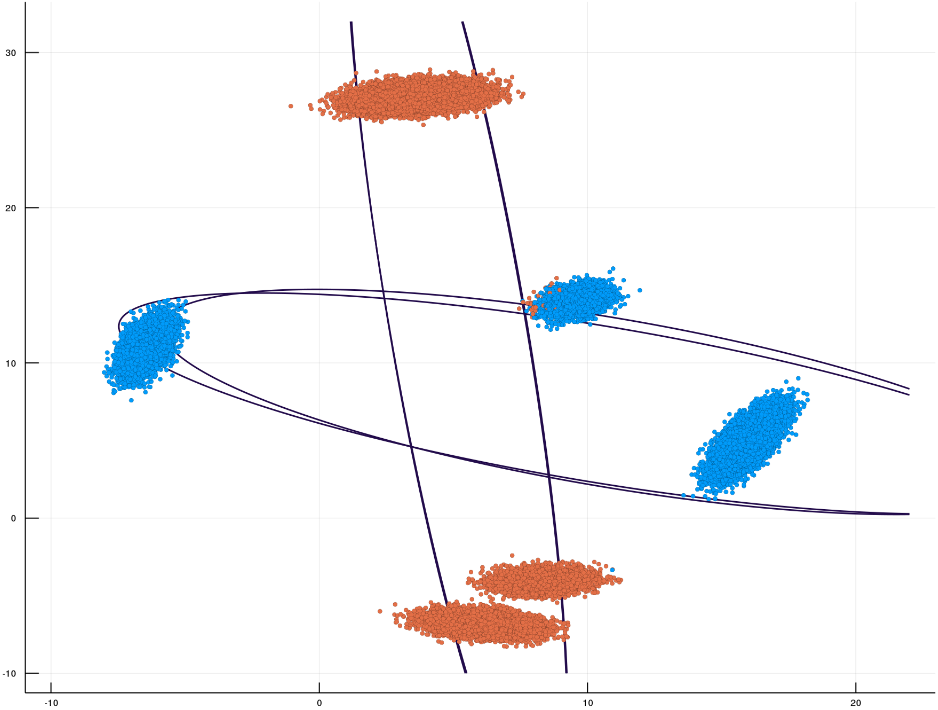

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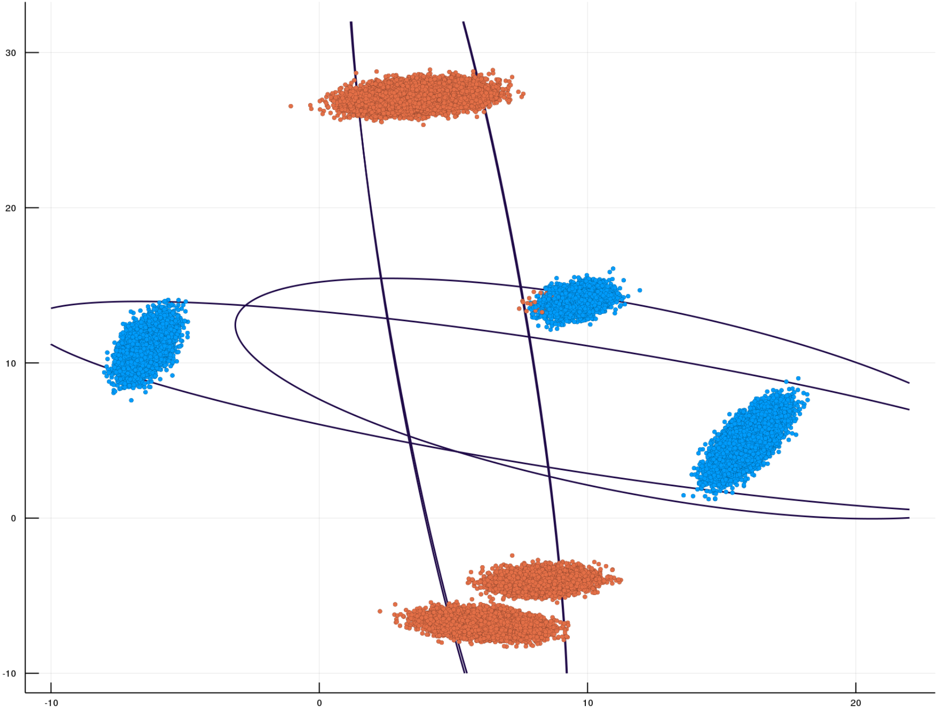

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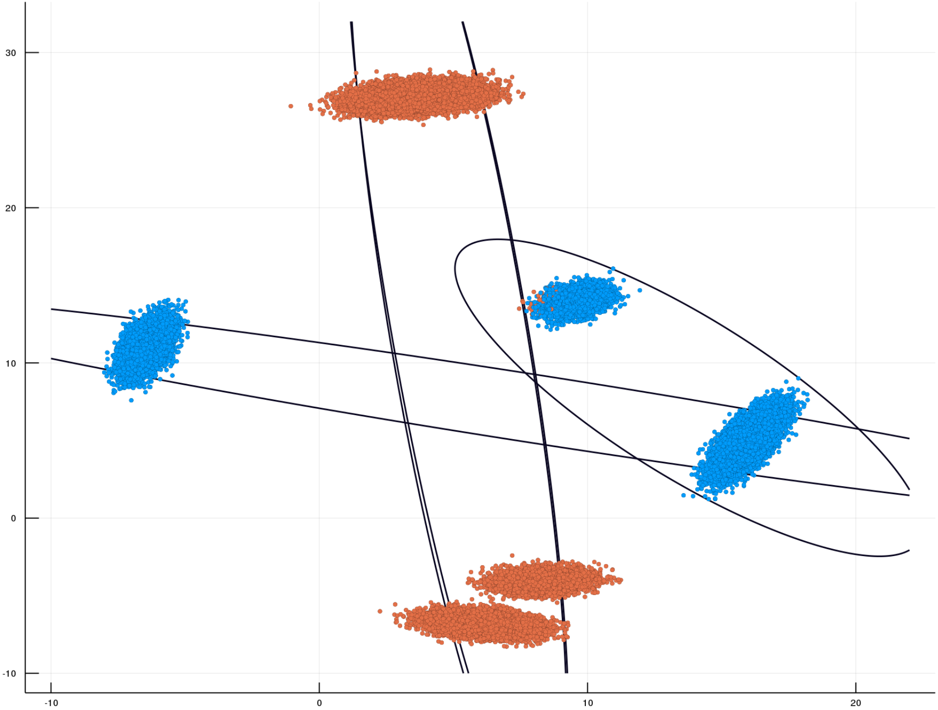
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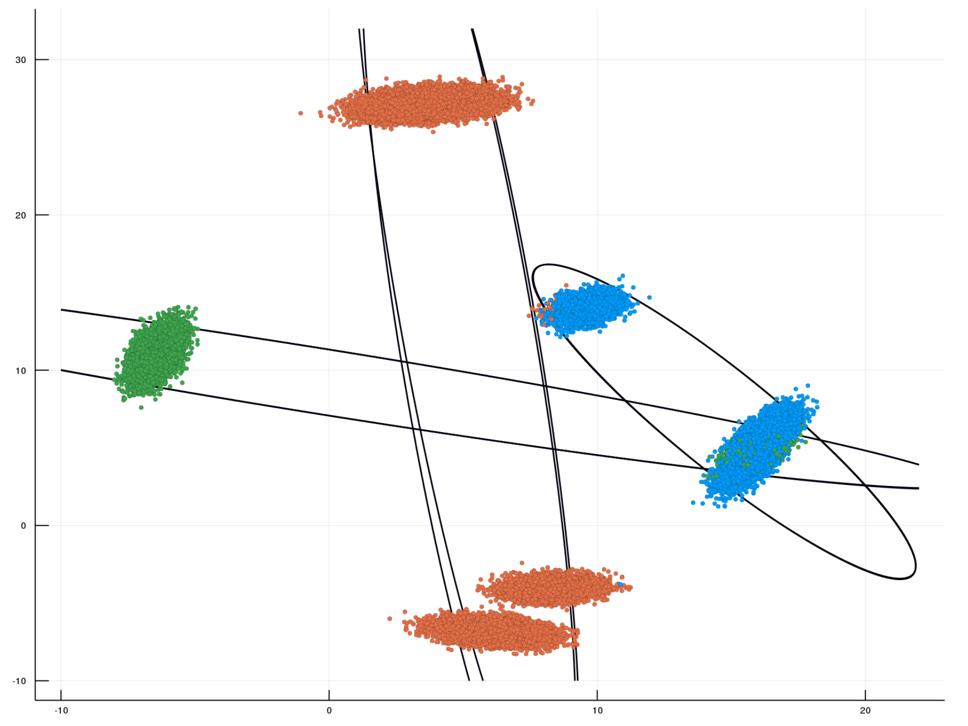

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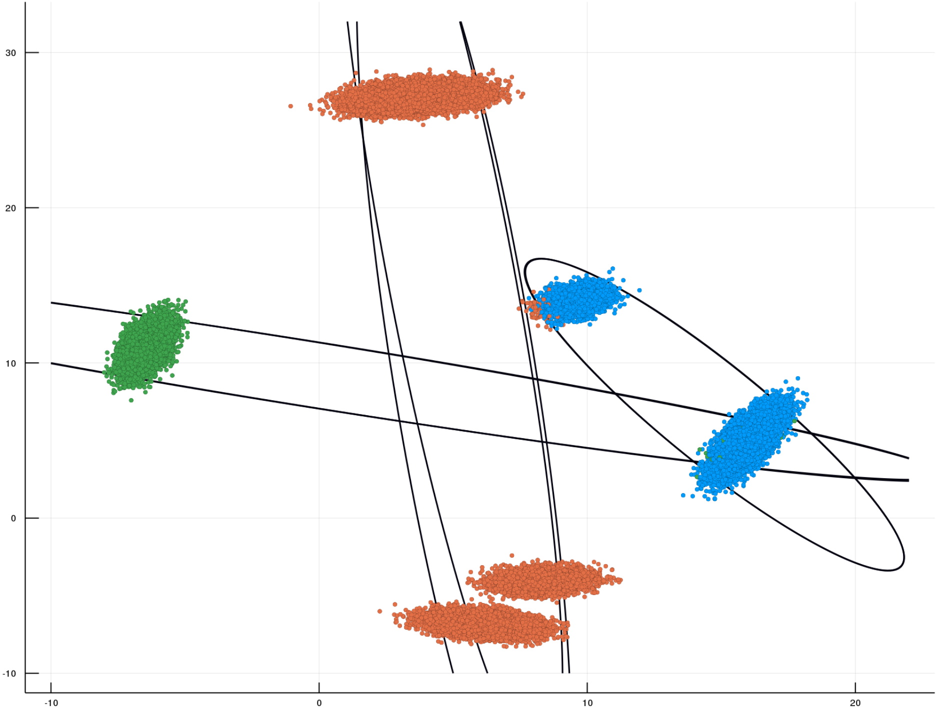
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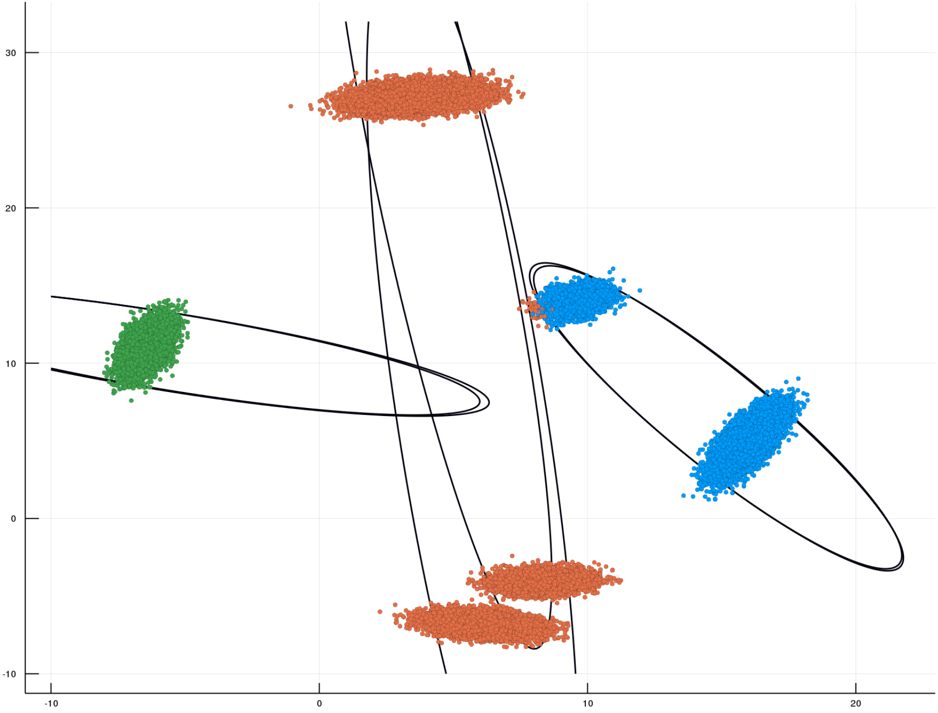

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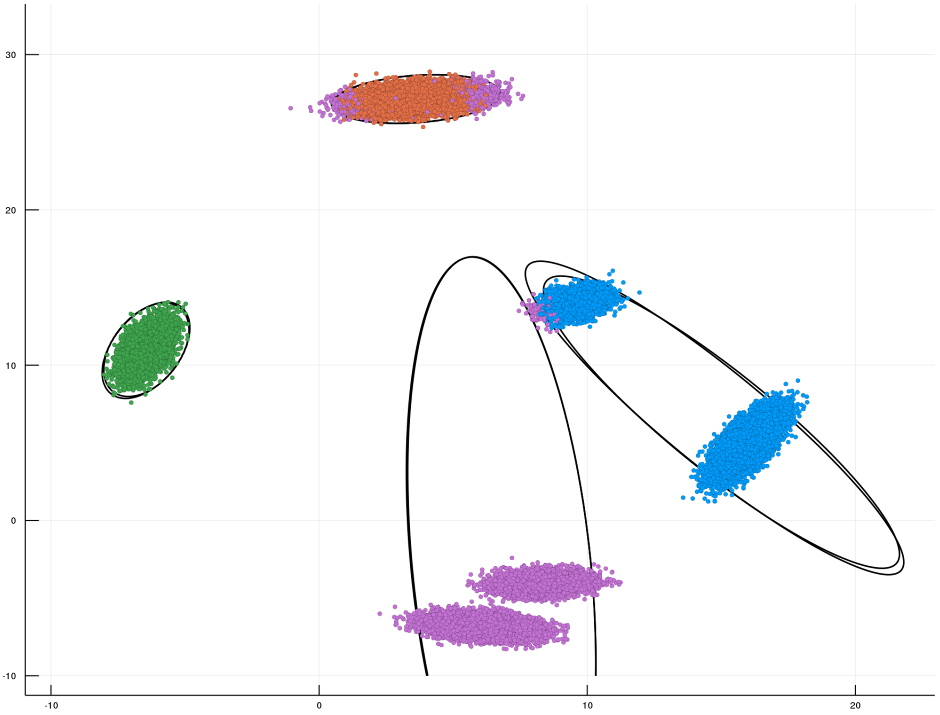

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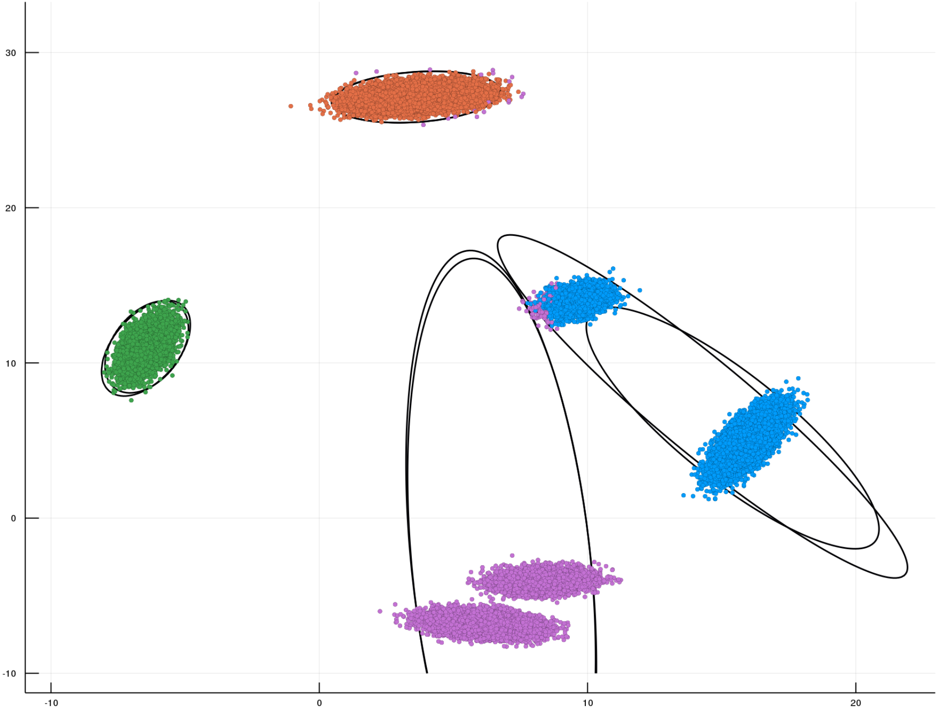
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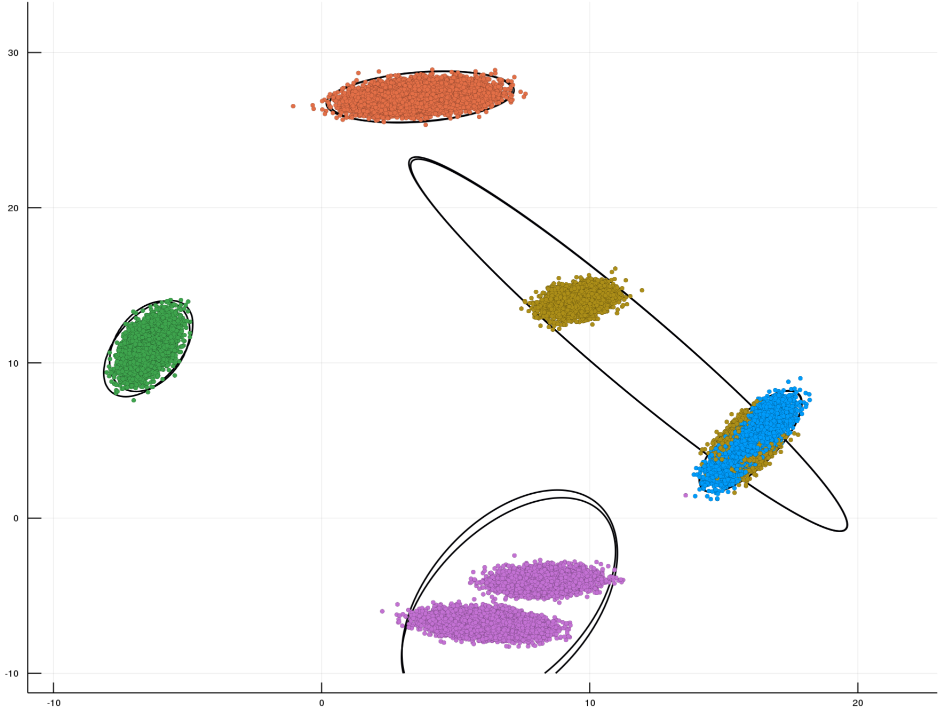

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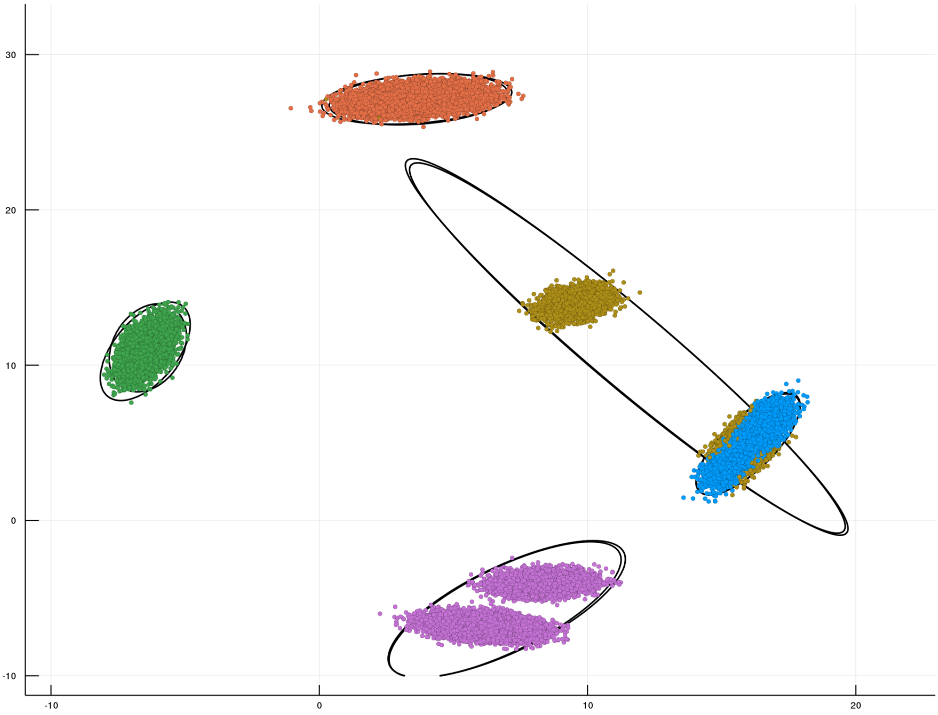
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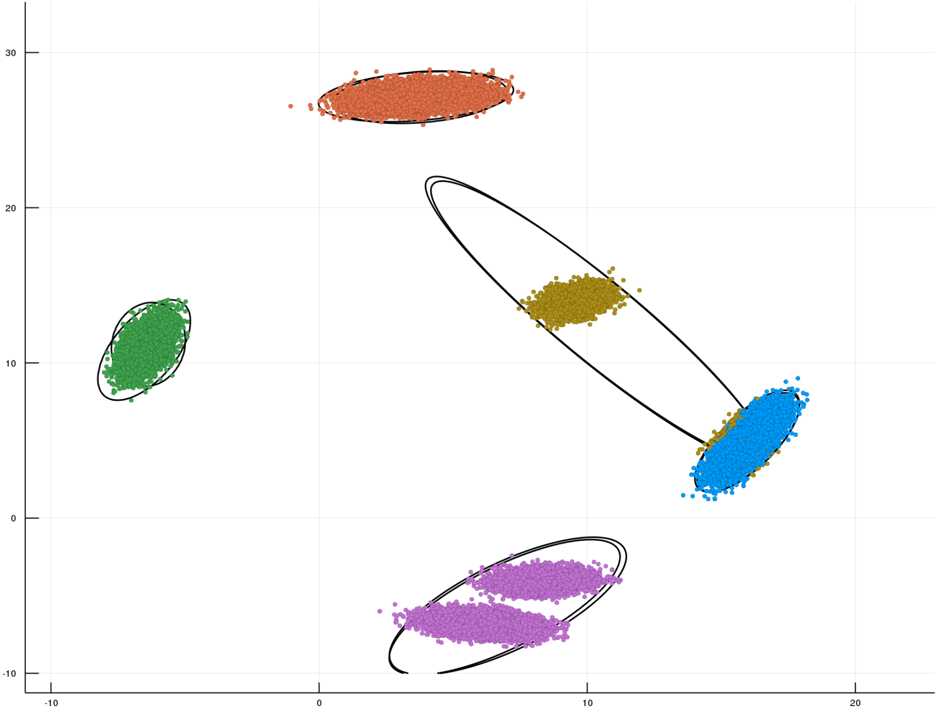

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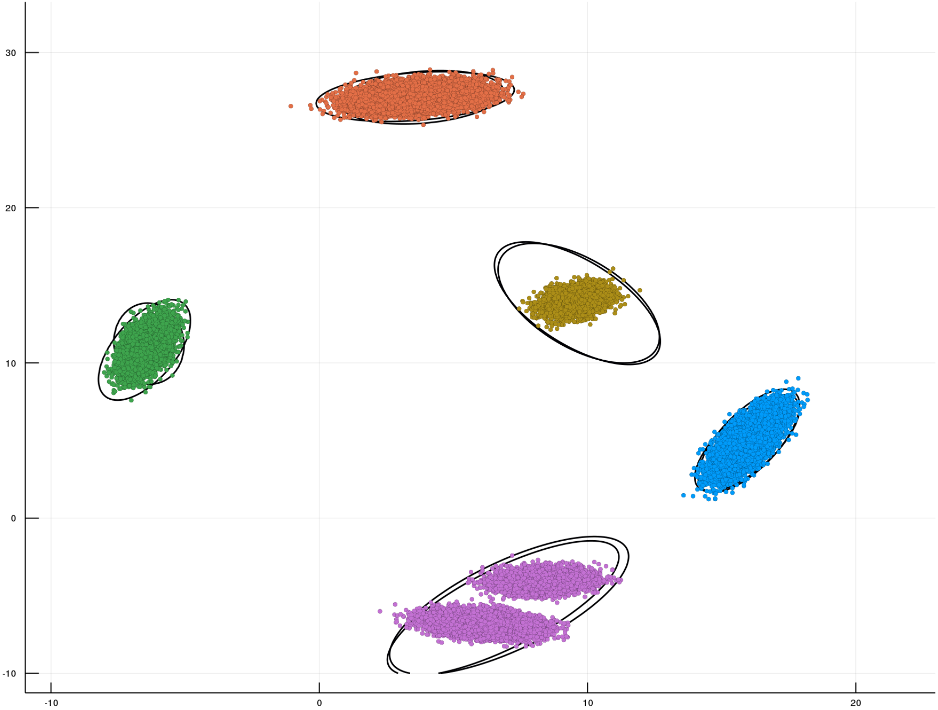
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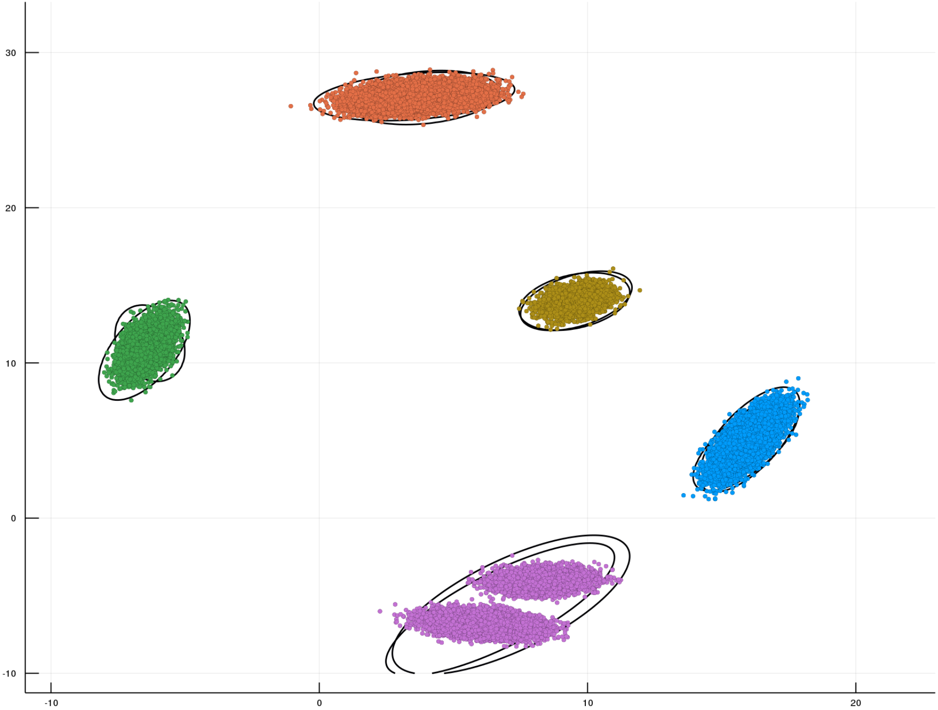
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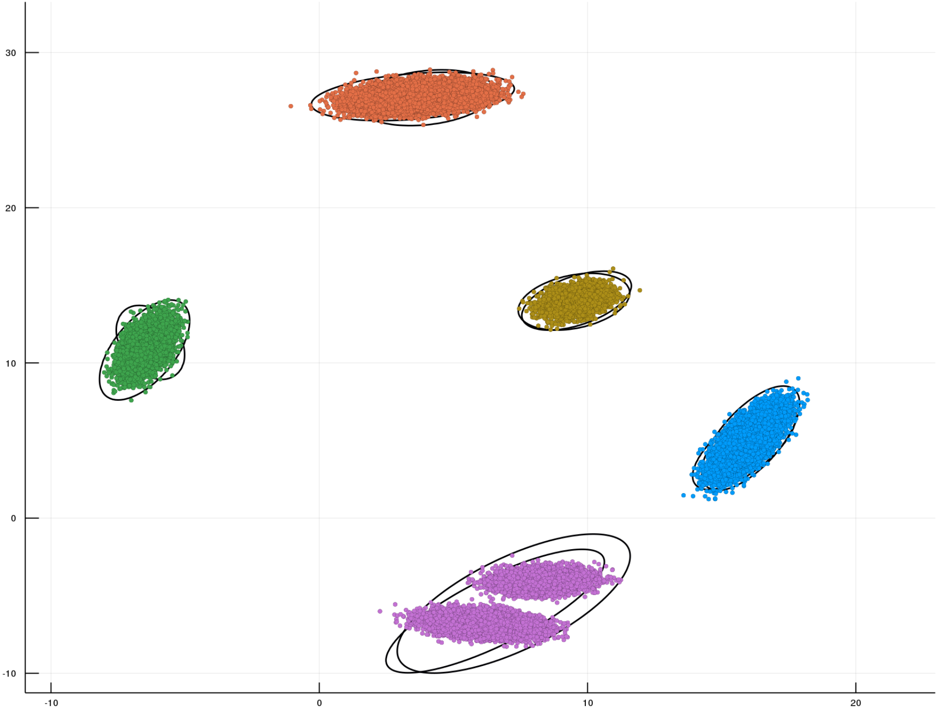
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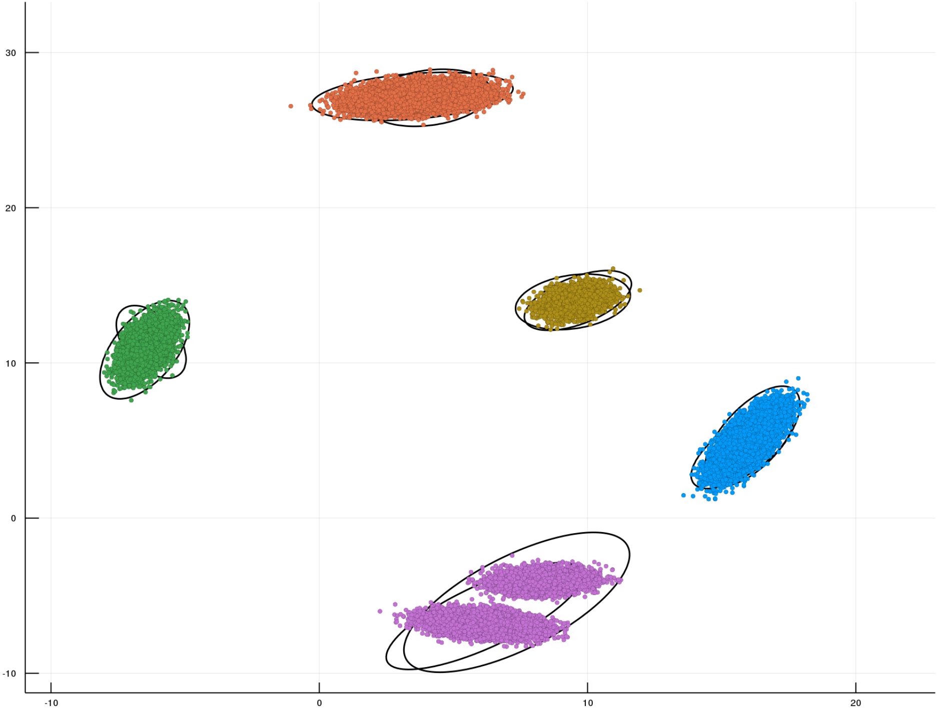

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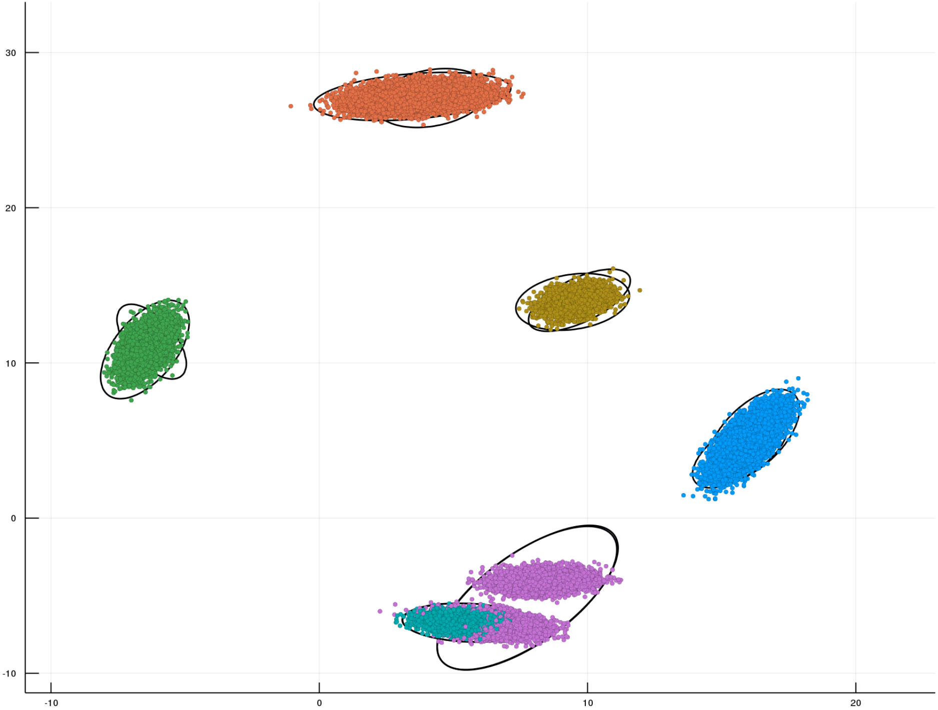

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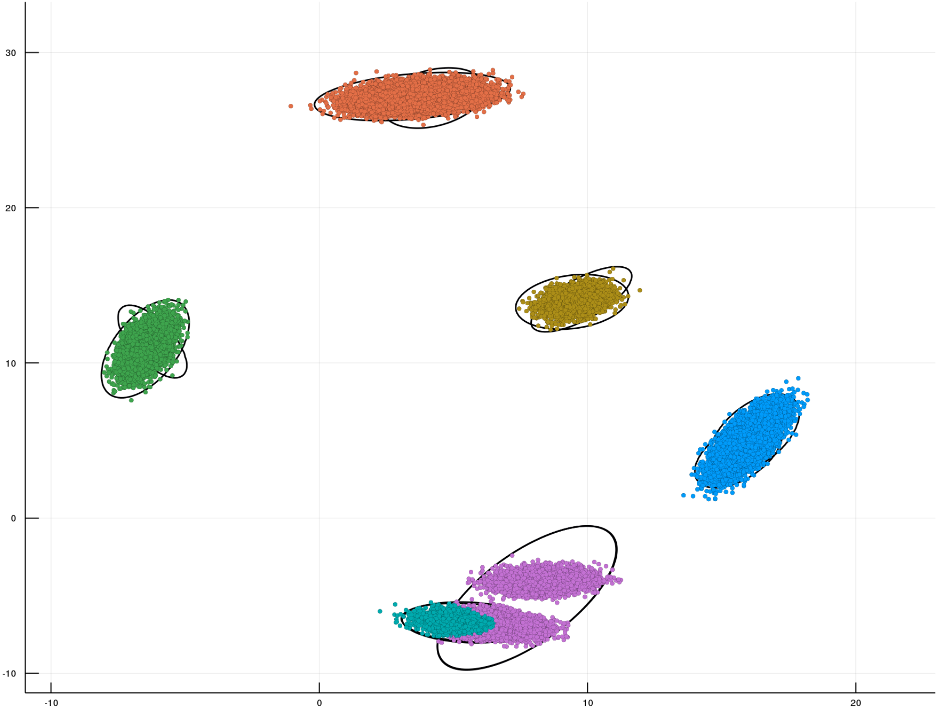

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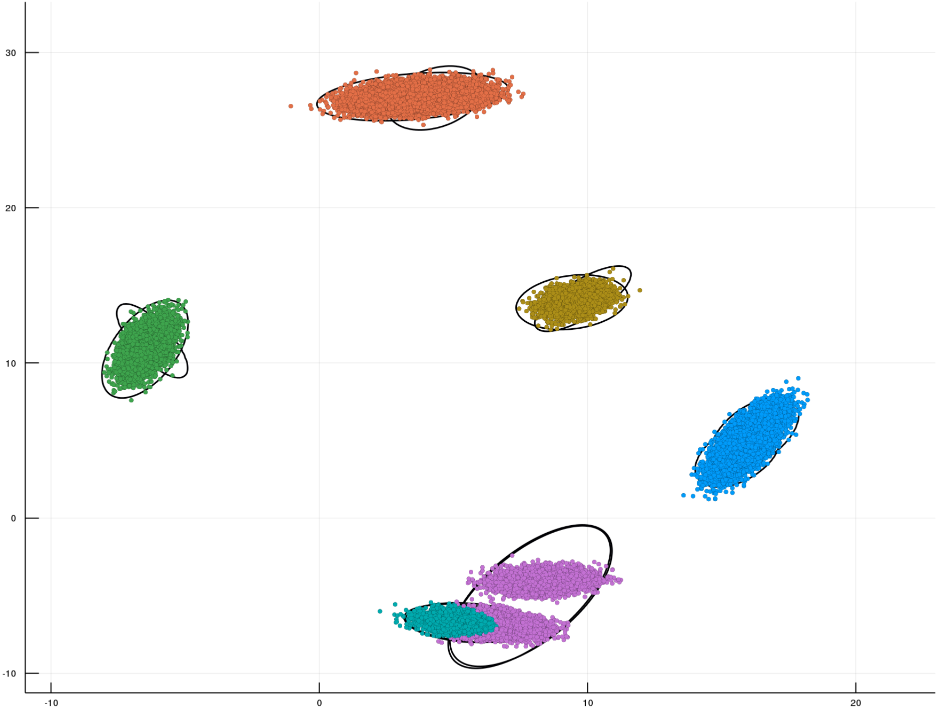
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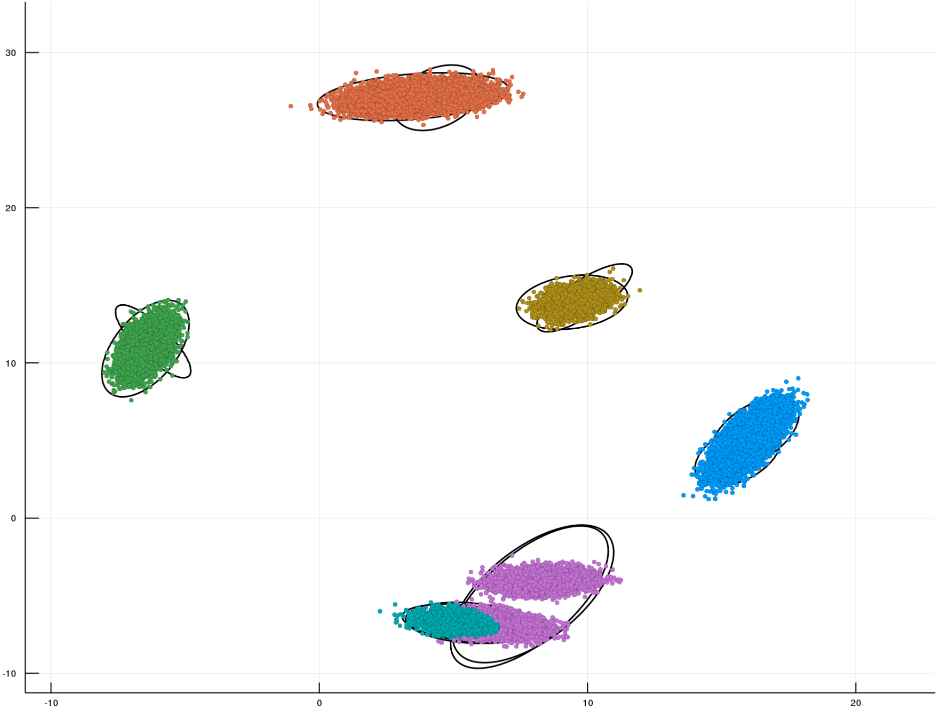

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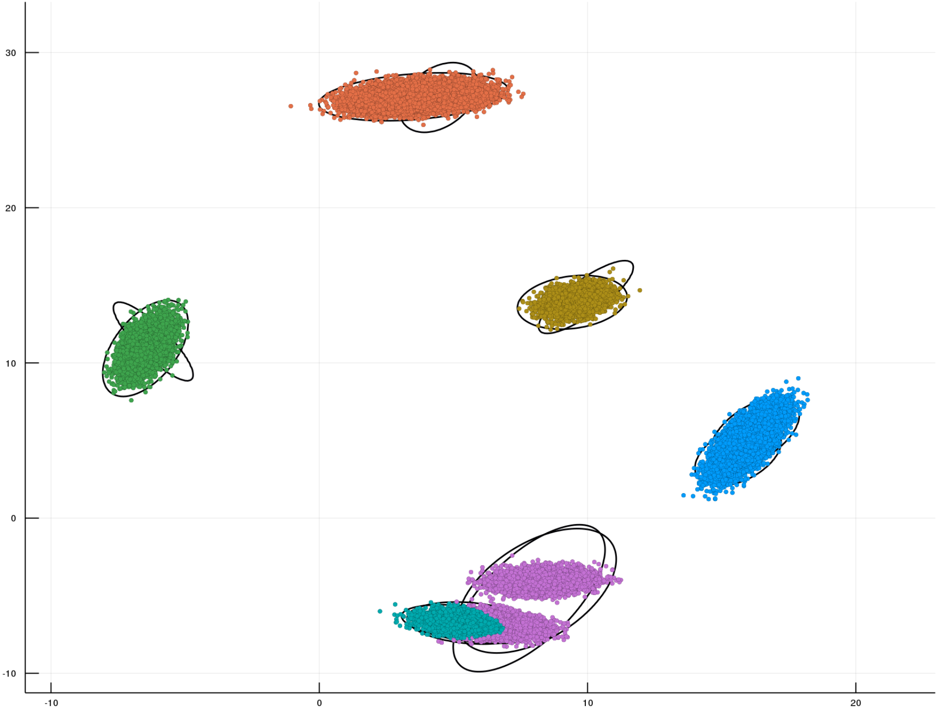
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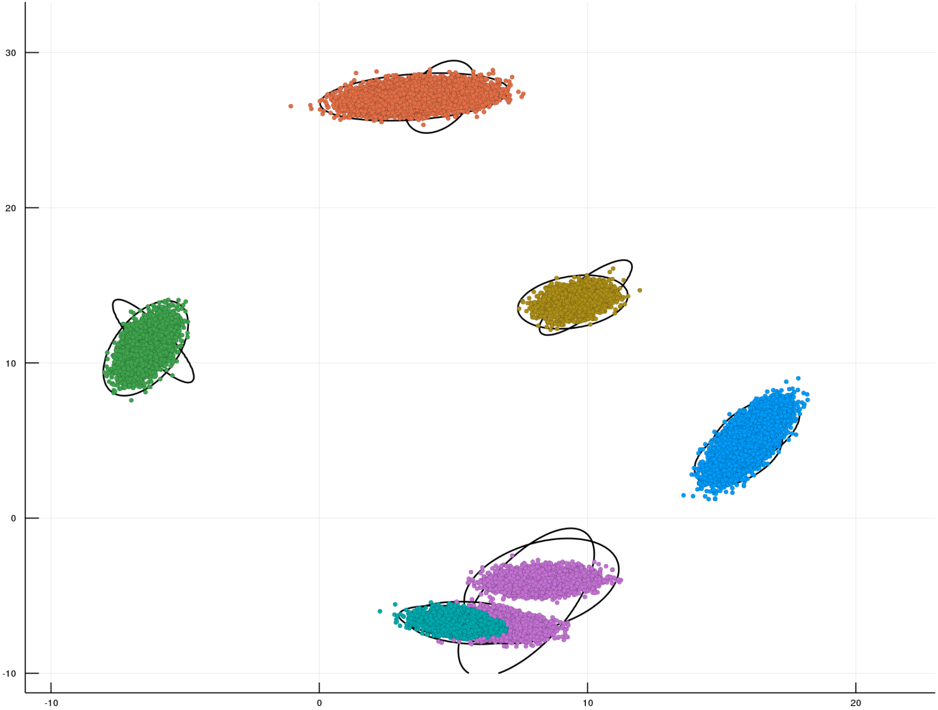
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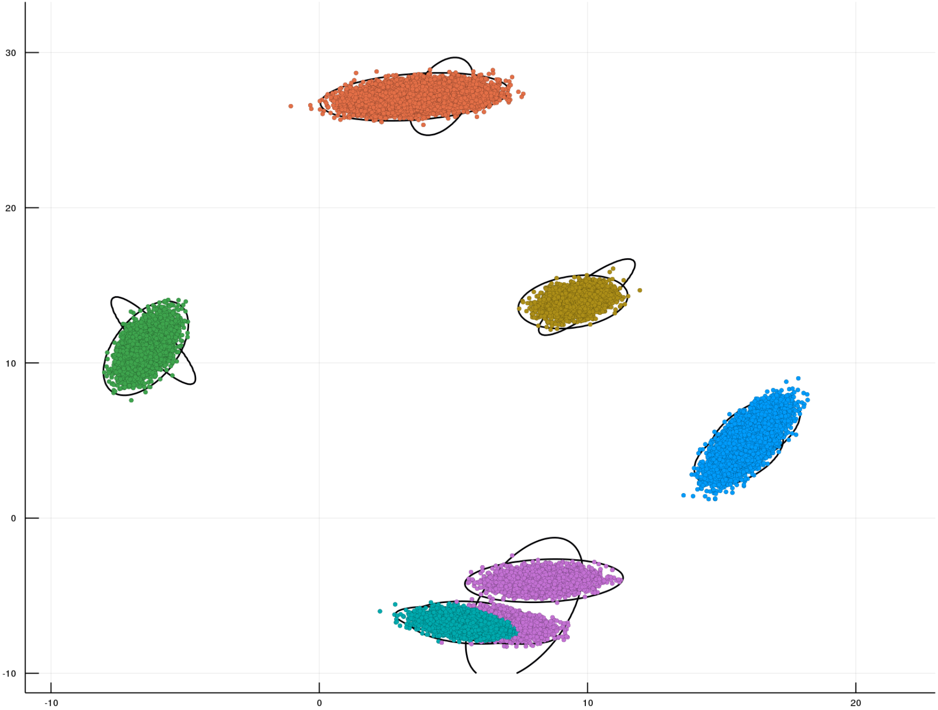
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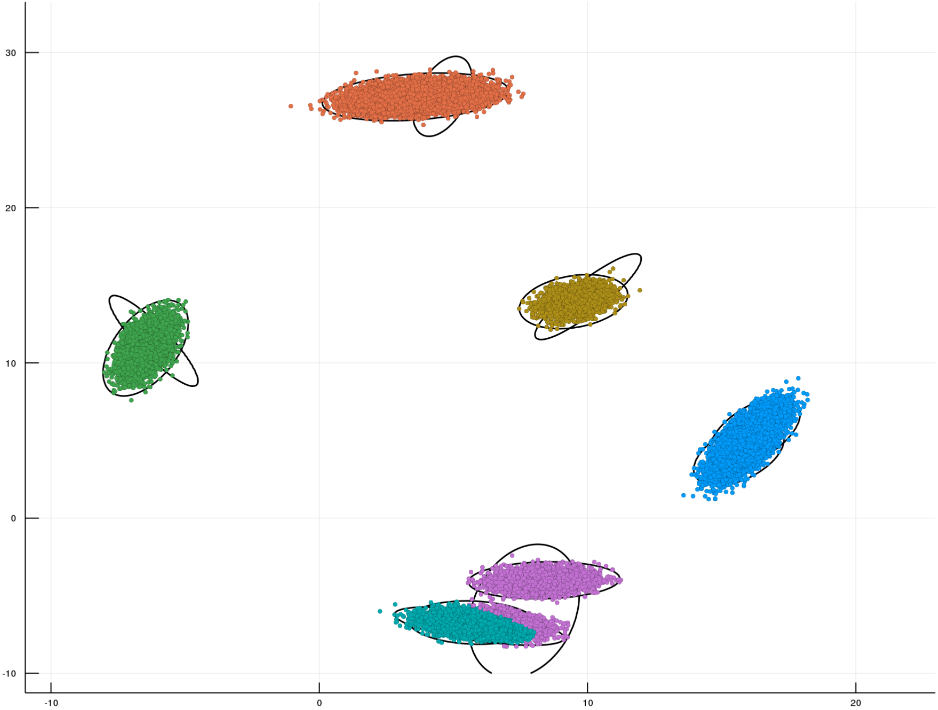
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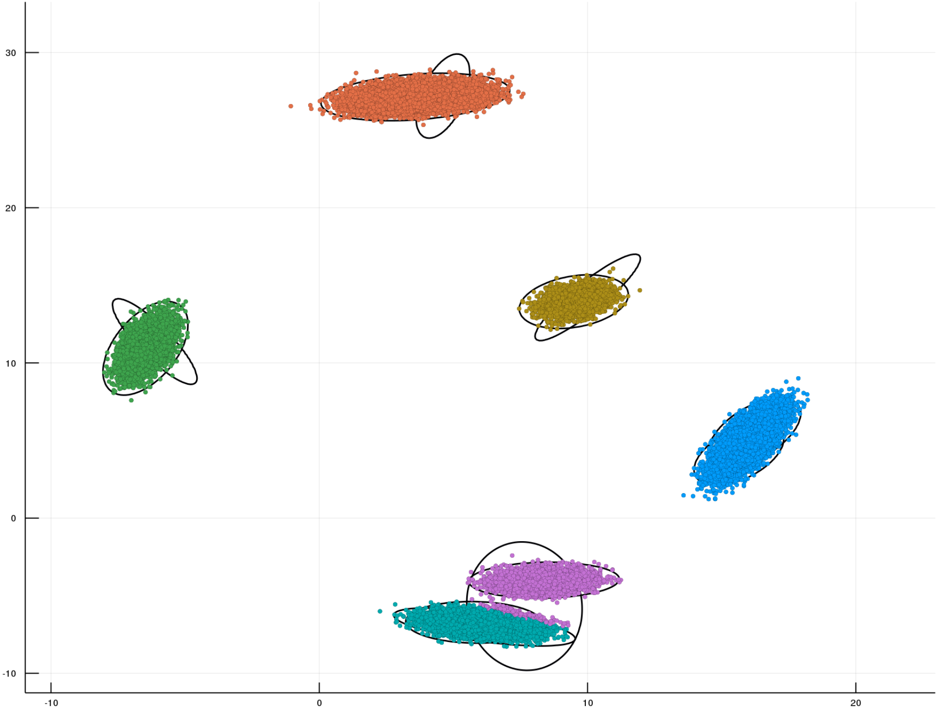

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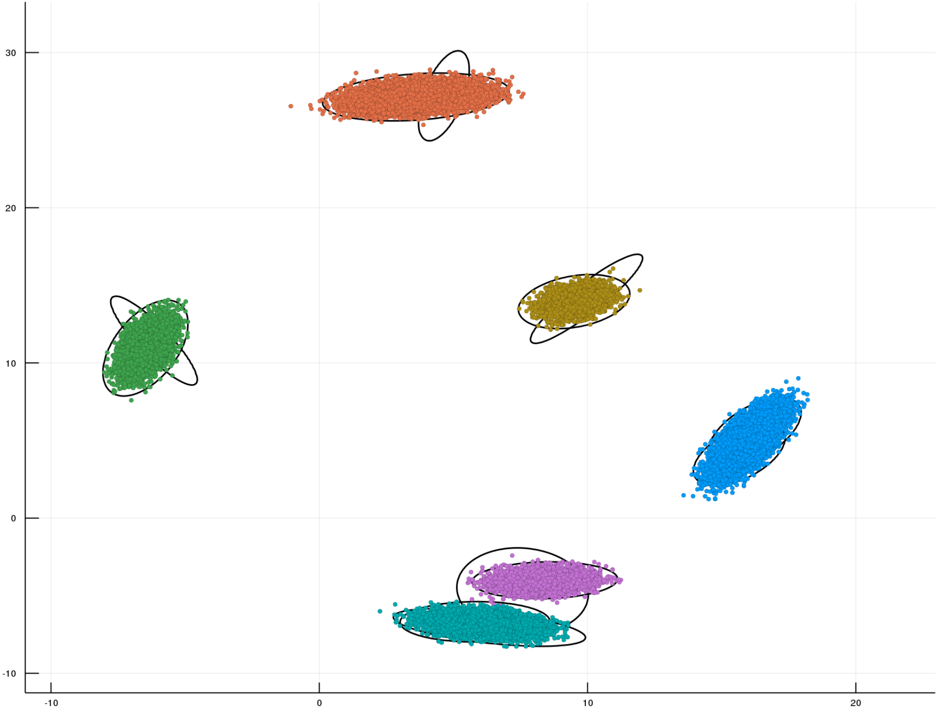
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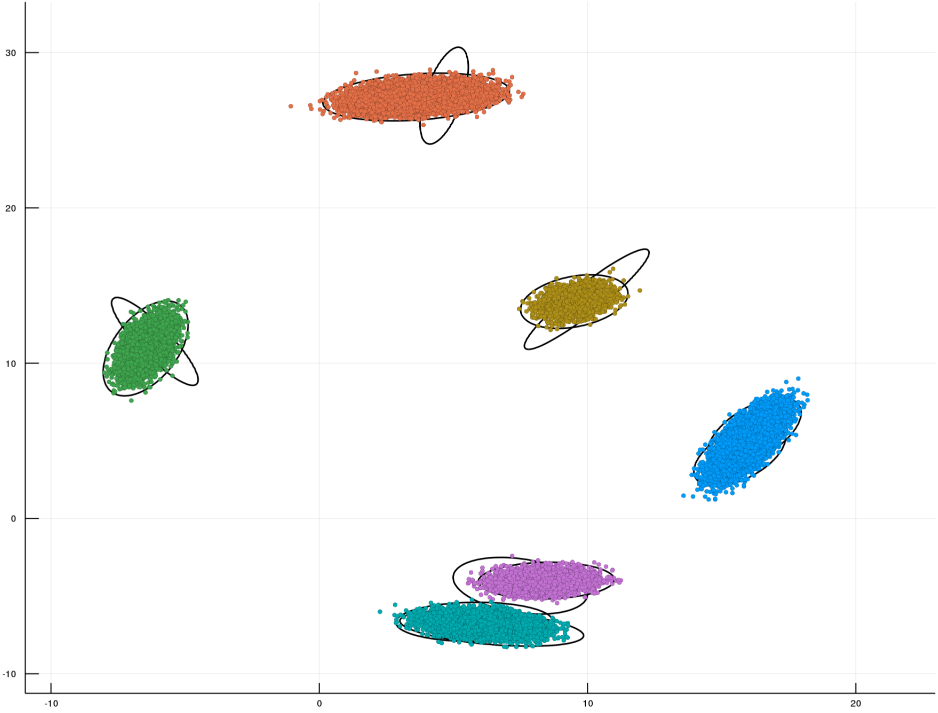
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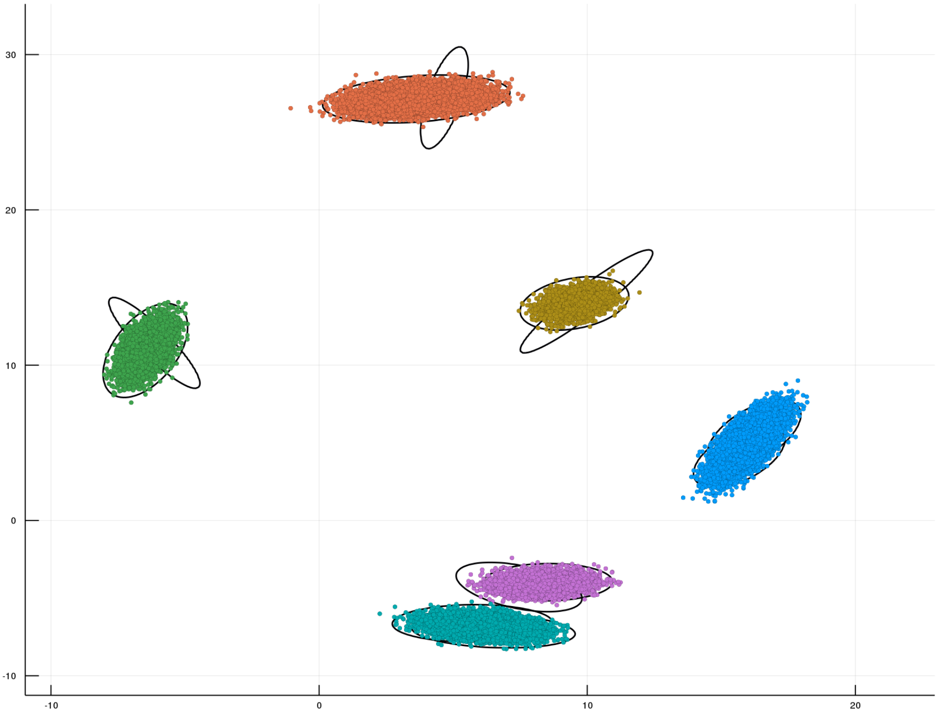

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  - **Easy** to distribute: the overhead, in terms of the **programmer's time**, for distributed computing is minimal.

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- At no point of time, a node can see the data which belongs to other nodes.

Master Node

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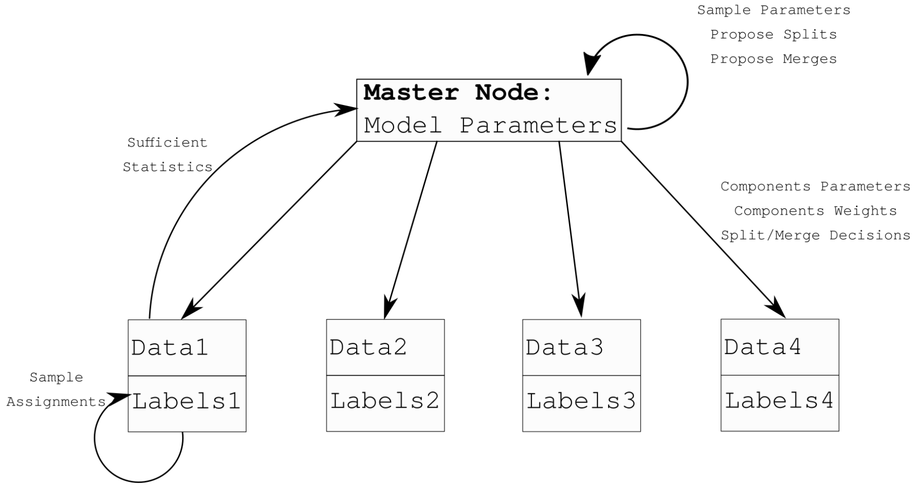
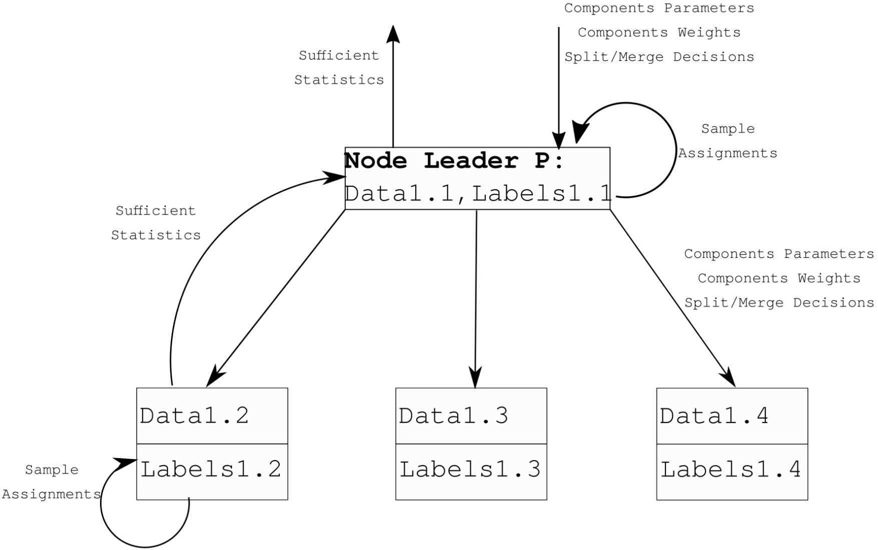
Architecture - Cluster



Architecture - Node



Implementation Key Ingredients

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```julia
abstract type distribution_hyper_params end
abstract type sufficient_statistics end
abstract type distibution_sample end
```

```julia
include("distributions/niw.jl")

random_seed = nothing

#Data Loading specifics
data_path = "/path/to/data/"
data_prefix = "data"

#Model Parameters
iterations = 32
hard_clustering = false
model_save_interval = 1000
initial_clusters = 1
total_dim = 2
α = 1.0

hyper_params = niw_hyperparams(1.0,
    zeros(total_dim),
    total_dim+3.0,
    Matrix{Float64}(I, total_dim, total_dim)*1.0)
```

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- 'Node Leader' can be turned off if required.

Results

For low-dimensional Gaussians: the previous method still wins

Cores $\times$ Machines	C++ [Chang & Fisher, NIPS '13]	Julia [this work]
1$\times$1	55.87	132.88
2$\times$1	35.48	78.28
4$\times$1	16.45	42.48
8$\times$1	**10.21**	32.95
8$\times$2	–	17.56
8$\times$3	–	16.73
8$\times$4	–	12.93

Table 1: Time (in [sec]) for running 100 DP-GMM iterations with $d = 2, N = 10^6, K = 6$.

Results

For high-dimensional Gaussians: the proposed method wins even when using only a single machine

Cores $\times$ Machines	C++ [Chang & Fisher, NIPS '13]	Julia [this work]
1×1	1637.52	416.40
2×1	720.29	232.62
4×1	480.50	139.86
8×1	262.41	94.64
8×2	–	53.01
8×3	–	39.30
8×4	–	**35.68**

Table 2: Time (in [sec]) for running 100 DP-GMM iterations of $d = 30, N = 10^6, K = 6$.

Conclusion based on our Perspective as ML Researchers

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- We don't claim that Julia is faster/better than X.
- Distributed implementations in Julia, ours included, offers a practical and monetary value due to the ease of development and abstraction level.
- We have extended the existing model, creating a fast, scalable, easy to use tool for DP-MM.
- The code will be available next month at: `https://github.com/dinarior/dpmm_subclusters.jl`

The Chinese Restaurant Process

Choosing a table for a new customer:

$$x_i | x_{-i} \sim CRP(\alpha, G_0) = \begin{cases} X_j & \frac{|X_{-i,j}|}{n-1+\alpha} \\ X_{K+1} \sim G_0 & \frac{\alpha}{n-1+\alpha} \end{cases} \quad (13)$$

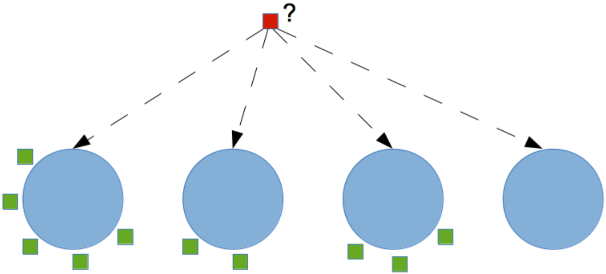
x_{-i} - All customers at the restaurant, excluding customer i

$|X_{-i,j}|$ - Customers count at table 1, excluding customer i.

α - Concentration parameter.

n - Customers count at the rest.

G_0 - Base probability measure.



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- Sample labels z for all points using:

$$z_i \sim DP - MM(\alpha, G_0) = \begin{cases} z_i = j & n_{-i,j} \cdot F_\theta(x_i|\theta_j) \\ z_i = K+1 & \alpha \cdot F_\theta(x_i|\theta_{K+1}) \end{cases} \tag{14}$$

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- Sample mixture components parameters conditioned on the current state of the model:

$$\theta_k | x,\ z,\ G_0 \tag{15}$$

References I

[1] J. Bezanson, A. Edelman, S. Karpinski, and V. Shah.
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